Ecuaciones Diferenciales Parabolicas

Ecuaciones Diferenciales Parciales

Parabolicas

$$i\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2}\right) + V(x, y, z)\Psi$$

Elipticas

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2}\right) + V(x, y, z)\Psi = E\Psi$$

Hiperbolicas

$$\frac{\partial^2 \Psi}{\partial t^2} = \left(\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2}\right)$$

Ejemplos : ED Parciales Elipticas

$$-\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\phi = S(x,y)$$

Potencial electrostatico

Densidad de carga

$$-\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\phi + V(x,y)\phi = \epsilon\phi$$

Funcion de onda

autovalores

potencial

Ejemplos : ED Parciales Parabolicas

Ec. Difusion

$$\frac{\partial \phi}{\partial t} = \nabla \cdot (D \nabla \phi) + S$$

Funcion fuente

Coeficiente de difusion

Resolucion Numerica

1D

$$\frac{\partial \phi'}{\partial t} = D \frac{\partial^2 \phi'}{\partial x'^2} + S'(x', t)$$

$$x = x' \cdot D^{-1/2}$$

$$x' = x \cdot D^{1/2}$$

$$\partial / \partial x' = (\partial x / \partial x') \partial / \partial x = D^{-1/2} \partial / \partial x$$

$$\partial^2 / \partial x'^2 = D^{-1} \partial^2 / \partial x^2$$

$$\frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial x^2} + S(x, t)$$

$$S(x, t) = S'(x \cdot D^{1/2}, t)$$

Resolucion Numerica: Discretizacion “Naïve”

$$\frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial x^2} + S(x,t)$$

Discretizacion espacial $h = 1/N$

Discretizacion temporal Δt

$$\frac{\phi_i^{n+1} - \phi_i^n}{\Delta t} = \frac{1}{h^2}\left(\delta^2 \phi^n\right)_i + S_i^n$$

$$f'' \approx \frac{1}{h^2}\left[f_1 - 2f_0 + f_{-1}\right]$$

$$\phi_{i+1}^n + \phi_{i-1}^n - 2\phi_i^n$$

Metodo Explicito

$$\phi_i^{n+1} = \phi_i^n + \left(\frac{1}{h^2}\left(\delta^2\right)\right)\phi_i^n \Delta t + S_i^n \Delta t$$

Operador que calcula la derivada segunda

Resolucion Numerica: Discretizacion "Naïve"

$$\phi_i^{n+1} = \phi_i^n + \left(\frac{1}{h^2} \left(\delta^2 \right) \right) \phi_i^n \Delta t + S_i^n \Delta t$$

$$\left(H \phi \right)_i \equiv - \frac{1}{h^2} \left(\delta^2 \phi \right)_i$$

$$\phi_i^{n+1} = \left(1 - H \ \Delta t \right) \phi_i^n + S_i^n \Delta t$$

Ejemplo practico de Discretizacion “Naïve”

Consideremos: $S = 0$

Condicion de contorno: $\phi(0) = \phi(1) = 0$

Condicion inicial: gaussiana centrada en x=1/2

$$\phi(x, t = 0) = e^{-20(x-1/2)^2} - \underbrace{e^{-20(x-3/2)^2} - e^{-20(x+1/2)^2}}$$

Aseguran las condiciones de contorno en:

$\phi(1)$ $\phi(0)$

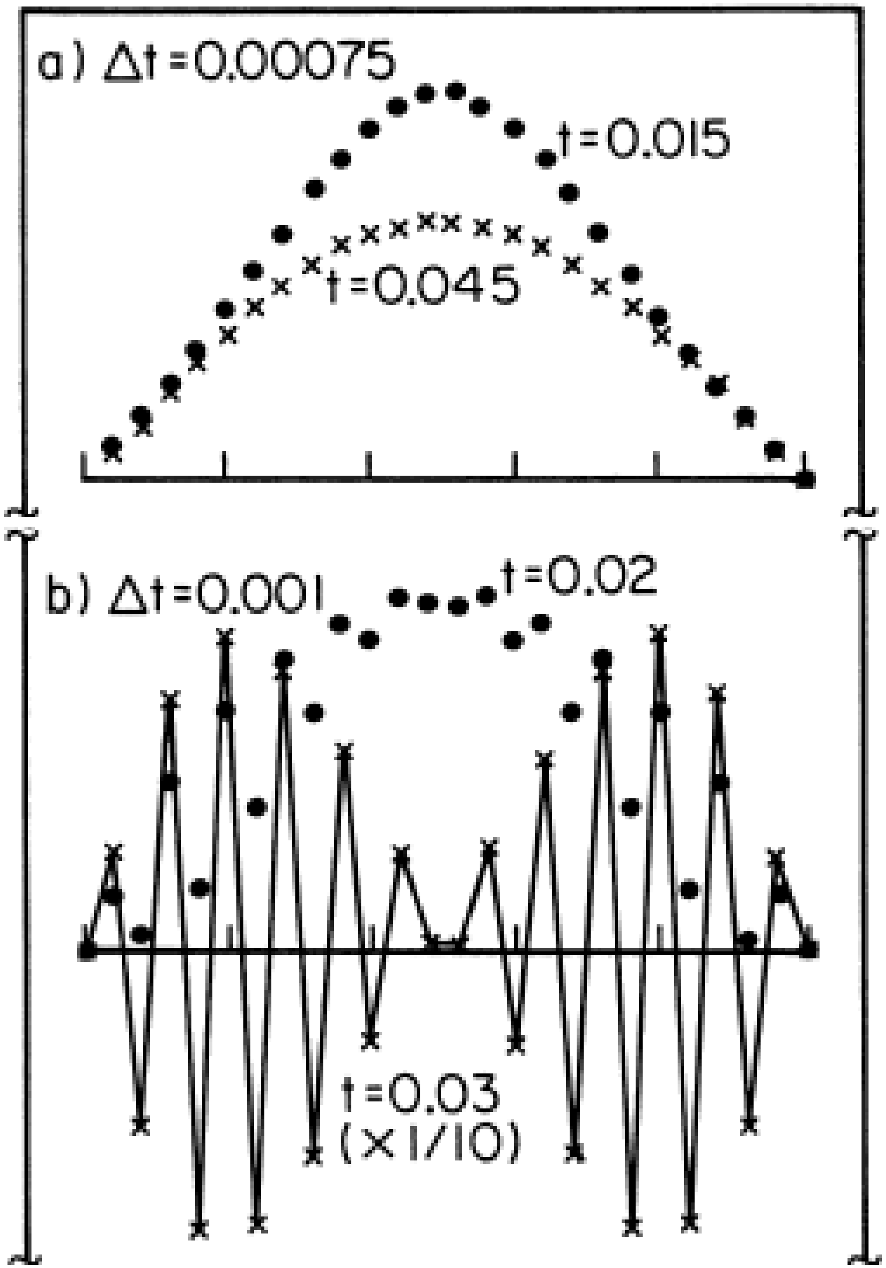
$$\phi_i^{n+1} = \phi_i^n + \left(\frac{1}{h^2}\left(\delta^2\right)\right)\phi_i^n \Delta t + S_i^n \Delta t$$

$$\phi_{i+1}^n + \phi_{i-1}^n - 2\phi_i^n$$

```
      PARAMETER (NSTEP=25)
      DIMENSION PHI(0:NSTEP)

      GAUSS(X,T)=EXP(-20.*(X-.5)**2/(1.+80*T))/SQRT(1+80*T)
      EXACT(X,T)=GAUSS(X,T)-GAUSS(X-1.,T)-GAUSS(X+1.,T)
      H=1./NSTEP
50    PRINT *, ' Enter time step and total time (0 to stop)'
      READ *,DT,TIME
      IF (DT .EQ. 0.) STOP
      NITER=TIME/DT
      DTH=DT/H**2
      T=0.
      PHI(0)=0.
      PHI(NSTEP)=0.
      DO 10 IX=1,NSTEP-1
         PHI(IX)=EXACT(IX*H,T)
10    CONTINUE
      DO 20 ITER=1,NITER
         POLD=0.
         DO 30 IX=1,NSTEP-1
            PNEW=PHI(IX)+DTH*(POLD+PHI(IX+1)-2*PHI(IX))
            POLD=PHI(IX)
            PHI(IX)=PNEW
30       CONTINUE
         IF (MOD(ITER,10) .EQ. 0) THEN
            PRINT *, ' iteration = ', ITER, ' time = ',ITER*DT
            T=ITER*DT
            DO 40 IX=1,NSTEP-1
               DIFF=PHI(IX)-EXACT(IX*H,T)
               PRINT *, ' phi = ', PHI(IX),' error = ', DIFF
40          CONTINUE
         END IF
20    CONTINUE
      GOTO 50
      END
```

$$\phi_{i+1}^n + \phi_{i-1}^n - 2\phi_i^n$$



a) Δt = 0.00075
t = 0.015
t = 0.045
b) Δt = 0.001
t = 0.02
t = 0.03
(×1/10)

Resolucion Numerica: Metodo Implicito

$$\phi_i^{n+1} = \phi_i^n + \left(\frac{1}{h^2}\left(\delta^2\right)\right)\phi_i^n \Delta t + S_i^n \Delta t$$

$$\frac{\phi_i^{n+1} - \phi_i^n}{\Delta t} = \frac{1}{h^2}\left(\delta^2 \phi^{n+1}\right)_i + S_i^n$$

$$\frac{\phi_i^{n+1} - \phi_i^n}{\Delta t} = \frac{1}{h^2}\left(\phi_{i+1}^{n+1} + \phi_{i-1}^{n+1} - 2\phi_i^{n+1}\right) + S_i^n$$